4.1 What is Probability?

Probability:

________________________ - how likely it is that an event will occur. When we use probability in a statement, we are using a number between 0 and 1 to indicate the likelihood of an event. We use the notation P(A) to determine the probability of event A. The closer to 1 the probability assignment is, the more likely the event is to occur.

*** Probabilities are always numbers __________________________________.
*** If an event is ___________________ to occur, the probability is 1, and if an event will certainly ___________ occur, then the probability is 0.

We need to learn how to find probabilities or assign them to events. We can use three major methods:

1) ___________________________________
2) ___________________________________
3) ___________________________________

1) __________________ - prediction based on previous outcomes.

Example: The New York Yankees will win many games this year.

2) __________________________________ - we have already discussed what relative frequency is when we looked at different types of histograms.

Probability Formula for Relative Frequency

Where *f* is the frequency of an event, and *n* is the sample size.

Example: What is the probability of selecting a female student in this class?

____

In the long run, as the sample size increases and increases, the relative frequency of outcomes get closer and closer to the theoretical (or actual) probability value.

An example of this is how the law of large numbers works is gambling at a casino.

If you flip a coin 10 times, what should the outcome be?

If you flip a coin 100 times, what should the outcome be?

If you flip a coin 1000 times, what should the outcome be?

Will this be true all of the time?

3) ________________________________ - when events have the same chance of happening.

Example: The probability of correctly guessing the answer to true-false questions.

Probability Formula When Outcomes Are Equally Likely

Can you think of any other situations where there are equally likely outcomes?

Sample Space

A statistical experiment (or an experiment) can be thought of as an activity that results in a definite outcome. Usually the outcome is in the form of a description, count, or measurement.

For example: If you toss a coin, there are only 2 possible outcomes (heads or tails).

______________________________ - set of all possible outcomes.

It is especially convenient to know the sample space where all outcomes are likely because then we can compute probabilities of various events using the following formula.

What is the sample space for each of the following?

Dice:

Cards:

Coins:

Spinners:

These will be used frequently throughout the chapter.

For a fair coin, find:

P(Heads) =

P(Tails) =

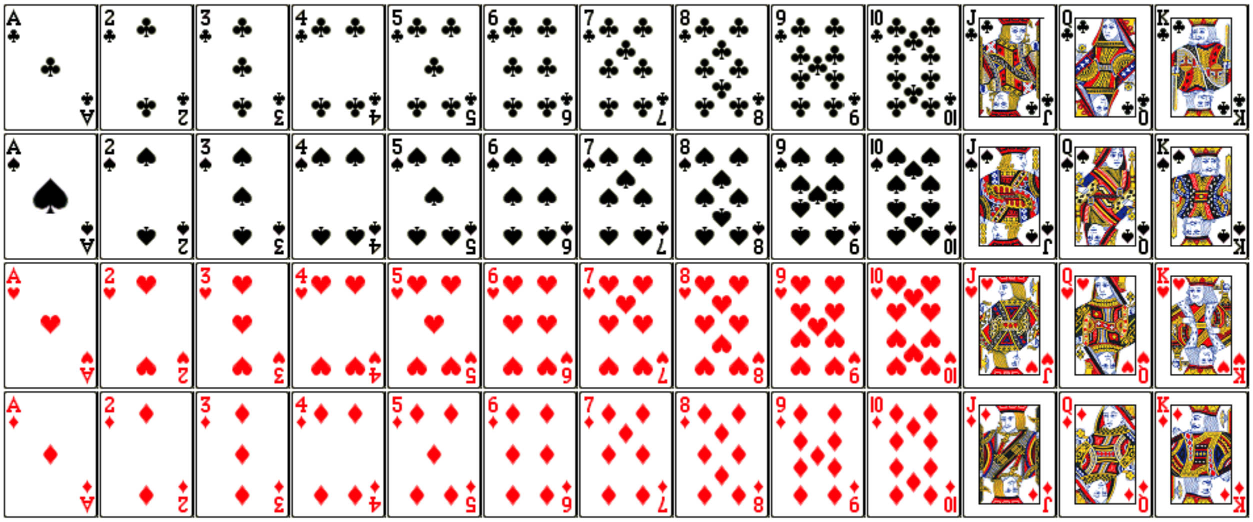
For a six-sided die, find:

P(4) =

P(even) =

P(Less than 5) =

Deck of Cards



________________________ of an Event

The sum of all the probabilities assigned to outcomes in a sample space must be 1. In that case, if you calculate your probability of an event occurring to be 0.55, then the probability of that event not occurring must be 0.45.For an event A, the event not A is called the complement of A. To compute the probability of the complement of A, use:

Five Important Facts about Probability

1) The probability of an event A is denoted by P(A).
2) The probability of any event is a number between 0 and 1. The closer to 1 the probability is, the more likely the event is.
3) The sum of the probabilities of outcomes in a sample space is 1.
4) Probabilities can be assigned by using three methods: intuition, relative frequency, or the formula for equally likely outcomes.
5) The probability that an event occurs plus the probability that the same event does not occur is 1.

Examples

1) If the probability that an event will occur is p, what is the probability that the event will not occur?

a) p	b) 1/p	c) p - 1	d) 1 - p

2) If the probability that an event will occur is x/4, what is the probability that the event will not occur?

a) (1 - x)/4	b) 4/x	c) (4 - x)/x	d) (4 - x)/4

3) If two fair dice are tossed once, the probability of getting 12 is 1/36. What is the probability of not getting 12?

a) 35/36	b) 6/36	c) 30/36	d) 34/36

4) On a test the probability of getting the correct answer to a certain question is represented by x/7. Which of the following cannot be a value of x?

a) -1	b) 1	c) 7	d) 0

5) When a number is chosen at random from the set {1,2,3,4,5,6}, which one of the following events has the greatest probability of occurring?

a) not choosing either 1 or 6	c) choosing a number greater than 3
b) choosing an even number	d) choosing a prime number

6) The probability of drawing a red marble from a sack of marbles is 2/5. Which one of the following sets of marbles could the sack contain?

a) 4 red marbles and 6 green marbles
b) 6 red marbles and 15 green marbles
c) 2 red marbles and 5 green marbles
d) 2 red marbles, 4 white marbles

7) A bag has five green marbles and four blue marbles. If one marble is drawn at random, what is the probability that it is not green?

a) 5/20 b) 1/9 c) 5/9 d) 4/9

8) A bag contains 2 red marbles and 3 blue marbles. If one marble is drawn at random, what is the probability that it is not green?

a) 3/5 b) 2/5 c) 4/5 d) 1/5

9) The footlights of a stage have 12 red bulbs, 8 blue bulbs, and 10 yellow bulbs. If all the bulbs are expected to last the same amount of time, what is the probability that a yellow bulb will burn out first?

a) 20/30 b) 10/20 c) 10/30 d) 1/30

10) During a half hour of television programming, eight minutes is used for commercials. If a television set is turned on at a random time during the half hour, what is the probability that a commercial is not being shown?

a) 8/30 b) 1 c) 22/30 d) 0

4.1 Homework

1) Suppose the newspaper states that the probability of rain today is 30%. What is the complement of the event "rain today"? What is the probability of the complement?

2) What is the probability of
(a) an event A that is certain to occur?

(b) an event B that is impossible?

3) What is the law of large numbers? If you were using the relative frequency of an event to estimate the probability of the event, would it be better to use 100 trials or 500 trials? Explain.

4) On a single toss of a fair coin, the probability of heads is 0.5 and the probability of tails is 0.5. If you toss a coin twice and get heads on the first toss, are you guaranteed to get tails on the second toss? Explain.

5) (a) If you roll a single die and count the number of dots on top, what is the sample space of all possible outcomes? Are the outcomes equally likely?

(b) Assign probabilities to the outcomes of the sample space of part (a). Do the probabilities add up to 1? Should they add up to 1? Explain.

(c) What is the probability of getting a number less than 5 on a single throw?

(d) What is the probability of getting 5 or 6 on a single throw?

6) A botanist has developed a new hybrid cotton plant that can withstand insects better than other cotton plants. However, there is some concern about the germination of seeds from the new plant. To estimate the probability that a seed from the new plant will germinate, a random sample of 3000 seeds was planted in warm, moist soil. Of these seeds, 2430 germinated.

(a) Use relative frequencies to estimate the probability that a seed will germinate. What is your estimate?

(b) Use relative frequencies to estimate the probability that a seed will *not* germinate. What is your estimate?

7) John runs a computer software store. Yesterday he counted 127 people who walked by his store, 58 of whom came into the store. Of the 58, only 25 bought something in the store.

(a) Estimate the probability that a person who walks by the store will enter the store.

(b) Estimate the probability that a person who walks into the store will buy something.

4.2 Some Probability Rules- Compound Events

_______________________________ - events where the occurrence or non-occurrence of one event does _____________ change the probability that the other will occur.

For example: You select a card at random, record it, and then place it back in the deck. Since you _______________ it, the probabilities when you select the 2nd card do not change. This is called ____________________. However, if you did not place the card back in the deck, the probabilities of your second selection would change.

1. Bag A contains 9 red marbles and 3 green marbles. Bag B contains 9 black marbles and 6 orange marbles. Find the probability of selecting one green marble from bag A and one black marble from bag B.

2. Two seniors, one from each government class are randomly selected to travel to Washington, D.C. Wes is in a class of 18 students and Maureen is in a class of 20 students. Find the probability that both Wes and Maureen will be selected.

3. If there was only one government class, and Wes and Maureen were in that class of 38 students, what would be the probability that both Wes and Maureen would be selected as the two students to go to Washington? Is this still an example of independent events?

______________________________ - when the outcome of the first event changes the probability of the next event.

4. A box contains 5 purple marbles, 3, green marbles, and 2 orange marbles. Two consecutive draws are made from the box **without replacement** of the first draw. Find the probability of each event.

a. P(orange first, green second)

b. P(both marbles are purple)

c. P(the first marble is purple, and the second is ANY color EXCEPT purple)

5. If you draw two cards from a standard deck of 52 cards **without replacement**, find:

a. P(King first, Jack second)

b. P(face card first, ace second)

c. P(2 aces)

Can you think of any examples that are independent or dependent?

Does the independence or dependence of an event matter?

Independence or dependence determines the way we compute probability of two events happening together.

For **Independent Events**, P(A and B) =

For **Dependent Events**, P(A and B) =

" " " , P(A and B) =

Probability of A or B

The condition A or B is satisfied by any of the following:

1) Any outcome of A occurs.
2) Any outcome of B occurs.
3) Any outcome in A and B occurs.

Example: If you want to compute the probability of drawing an ace or a king on 2 consecutive cards, you would just add the two probabilities together.

It is important to distinguish between the **"or"** combinations and the **"and"** combinations because we apply different rules to compute the probabilities.

Probability Rules: (GENERAL)

"OR" Problems -

"AND" Problems -

Examples:

1) The probability of throwing two fours on a single toss of a pair of dice is
a) 1/6 b) 1/3 c) 1/12 d) 1/36

2) If two coins are tossed the probability of getting two tails is
a) 1/8 b) 1/3 c) 1/4 d) 1/2

3) If two cards are drawn from a standard deck of 52 cards without replacement, what is the probability that both cards are fives?
a) $4/52 \cdot 3/52$ b) $5/52 \cdot 4/51$ c) $1/4 \cdot 1/3$ d) 2/52

4) From a deck of 52 cards, two cards are randomly drawn without replacement. What is the probability of drawing two hearts?
a) $13/52 \cdot 12/51$ b) $13/52 \cdot 13/51$ c) 2/52 d) $13/52 \cdot 13/51$

5) If two cards are drawn from a standard deck of 52 cards without replacement, what is the probability that both cards will be black aces?
a) $2/52 \cdot 2/51$ b) $4/52 \cdot 3/51$ c) $4/52 \cdot 4/51$ d) $2/52 \cdot 1/51$

6) If 2 cards are dealt randomly from a standard deck of 52 cards, what is the probability that they are both red queens?
a) $2/52 \cdot 1/51$ b) 2/26 c) $4/52 \cdot 31/51$ d) 2/52

7) From a standard deck of 52 cards, two cards are drawn at random without replacement. What is the probability that both cards drawn are aces?
a) 12/2,652 b) 4/2,652 c) 4/52 d) 6/2,652

8) A gumball machine contains six yellow gumballs and five orange gumballs. What is the probability of obtaining, at random and without replacement, two yellow gumballs?
a) 30/110 b) 36/121 c) 30/121 d) 36/110

9) A bag of marbles contains three blue, one black, and four yellow marbles. If two marbles are chosen at random without replacement, what is the probability that both marbles will be yellow?
a) 3/14 b) 7/56 c) 1/3 d) 1/4

10) A pencil holder contains only six blue pencils and three red pencils. If two pencils are drawn at random, what is the probability both are blue?

a) 6/9 b) 30/72 c) 2/9 d) 30/81

Check for Dependence

If two events are ____________________ then

P(A) * P(B) = P(A and B)

Determine if events A and B are independent.

$P(A) = \frac{2}{5}$ $P(B) = \frac{1}{5}$ $P(A \text{ and } B) = \frac{2}{25}$

$P(A) = \frac{2}{5}$ $P(B) = \frac{1}{4}$ $P(A \text{ and } B) = \frac{1}{25}$

$P(A) = \frac{9}{20}$ $P(B) = \frac{1}{2}$ $P(A|B) = \frac{27}{50}$

$P(\text{not } A) = \frac{3}{4}$ $P(B) = \frac{3}{10}$ $P(A \text{ and } B) = \frac{3}{40}$

Conditional Probability

- If events are ______________________, the occurrence of one event changes the probability of the other.
- The notation P(A|B) is read _____________________________________
- P(A, given B) equals the probability that event A occurs, assuming that B has _______________ occurred.

Suppose an employee is selected at random from the 140 Hopewell employees. Let us use the following notation to represent different events of choosing: E = executive; PW = production worker; D = Democrat; R = Republican; I = Independent.

(a) Compute P(D) and P(E).

(b) Compute $P(D \mid E)$.

(c) Are the events D and E independent?

(d) Compute $P(D \text{ and } E)$.

____ - events that cannot occur together.

- events that do ___________ have any outcomes in common.
- P(A and b) = 0

For mutually exclusive events A and B, use:

If the events are not mutually exclusive, we use a more general formula, which is the addition rule for any events A and B.

***If you are unsure as to which formula to use, always use the 2nd formula. This will give you the correct answer regardless.

Examples:

	Political Affiliation			
Employee Type	Democrat (*D*)	Republican (*R*)	Independent (*I*)	Row Total
Executive (*E*)	5	34	9	48
Production worker (*PW*)	63	21	8	92
Column Total	68	55	17	140 Grand Total

Find:

P(I) =

P(PW) =

P(I and PW) =

P(I | PW) =

Are the events I and PW independent?

Are the events I and PW mutually exclusive?

The Cost Less Clothing Store carries seconds in slacks. If you buy a pair of slacks in your regular waist size without trying them on, the probability that the waist will be too tight is 0.30 and the probability that it will be too loose is 0.10.

(a) Are the events too tight or too loose mutually exclusive?

(b) If you choose a pair of slacks at random in your regular waist size, what is the probability that the waist will be too tight or too loose?

Professor Jackson is in charge of a program to prepare people for a high school equivalency exam. Records show that 80% of the students need work in math, 70% need work in English, and 55% need work in both areas.

(a) Are the events needs math and needs English mutually exclusive?

(b) Use the appropriate formula to compute the probability that a student selected at random needs math *or* needs English.

Combination of Several Events

The addition rule for mutually exclusive events can be ____________________ so that it applies to the situation in which we have more than two events that are mutually exclusive to all other events.

Laura is playing Monopoly. On her next move she needs to throw a sum bigger than 8 on the two dice in order to land on her own property and pass GO. What is the probability that Laura will roll a sum bigger than 8?

A statistical experiment or statistical observation is any random activity that results in a recordable outcome. The sample space is the set of all simple events that are the outcomes of the statistical experiment and cannot be broken into other "simpler" events. A general event is any subset of the sample space. The notation $P(A)$ designates the probability of event A.

1. $P(\text{entire sample space}) = 1$
2. For any event A: $0 \leq P(A) \leq 1$
3. A^c designates the complement of A: $P(A^c) = 1 - P(A)$
4. Events A and B are independent events if $P(A) = P(A \mid B)$.
5. Multiplication Rules

   General: $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

   Independent events: $P(A \text{ and } B) = P(A) \cdot P(B)$
6. Conditional Probability: $P(A \mid B) = \dfrac{P(A \text{ and } B)}{P(B)}$
7. Events A and B are mutually exclusive if $P(A \text{ and } B) = 0$.
8. Addition Rules

   General: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

   Mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$

4.2 Homework

1) You roll two fair dice, a green one and a red one.
(a) Are the outcomes on the dice independent?

(b) Find P(5 on green die *and* 3 on red die).

(c) Find P(3 on green die *and* 5 on red die).

(d) Find P((5 on green die *and* 3 on red die) *or* (3 on green die *and* 5 on red die)).

2) You roll two fair dice, a green one and a red one.
(a) Are the outcomes on the dice independent?

(b) Find P(1 on green die *and* 2 on red die).

(c) Find P(2 on green die *and* 1 on red die).

(d) Find P((1 on green die *and* 2 on red die) *or* (2 on green die *and* 1 on red die)).

3) You roll two fair dice, a green one and a red one.
(a) What is the probability of getting a sum of 6?

(b) What is the probability of getting a sum of 4?

(c) What is the probability of getting a sum of 6 *or* 4? Are these outcomes mutually exclusive?

4) You draw two cards from a standard deck of 52 cards without replacing the first one before drawing the second.
(a) Are the outcomes on the two cards independent? Why?

(b) Find P(ace on 1st card *and* king on 2nd).

(c) Find P(king on 1st card *and* ace on 2nd).

5) You draw two cards from a standard deck of 52 cards without replacing the first one before drawing the second.
(a) Are the outcomes on the two cards independent? Why?

(b) Find P(3 on 1st card *and* 10 on 2nd).

(c) Find P(10 on 1st card *and* 3 on 2nd).

(d) Find the probability of drawing a 10 *and* a 3 in either order.

6) You draw two cards from a standard deck of 52 cards, but before you draw the second card, you put the first one back and reshuffle the deck.
(a) Are the outcomes on the two cards independent? Why?

(b) Find P(3 on 1st card *and* 10 on 2nd).

(c) Find P(10 on 1st card *and* 3 on 2nd).

(d) Find the probability of drawing a 10 *and* a 3 in either order.

7)

	Female	Male	Total
Will Graduate	60	14	
Will Not Graduate	25	2	
Total			

(a) P(student will graduate | student is female).

(b) P(student will graduate *and* student is female).

(c) P(student will graduate | student is male).

(d) P(student will graduate *and* student is male).

(e) P(student will graduate).

(f) The events described by the phrases "will graduate *and* is female" and "will graduate, *given* female" seem to be describing the same students. Why are the probabilities P(will graduate *and* is female) and P(will graduate | female) different?

4.3 Trees and Counting Techniques

1. Tree Diagrams
2. Multiplication Rule of Counting
3. Permutations
4. Combinations

_____________________________ - a method of listing outcomes of an experiment consisting of a series of activities

Tree diagram for the experiment of tossing two coins:

Suppose there are five balls in an urn. They are identical except for color. Three of the balls are red and two are blue. You are instructed to draw out one ball, note its color, and set it aside. Then you are to draw out another ball and note its color. What are the outcomes of the experiment? What is the probability of each outcome?

Counting Techniques:

If we are only interested in the number of outcomes created by a series of events, the multiplication rule will give us the total number of outcomes more directly.

If there are n possible outcomes for event E_1 and m possible outcomes for event E_2, then there are a total of n times m or nm possible outcomes for the series of events E_1 followed by E_2.

This rule extends to outcomes created by a series of three, four, or more events. Just simply multiply the number of events to get the total number of outcomes for the series.

1. The local pizzeria offers a choice of 2 pizzas - supreme or vegetarian, 3 sides - chips, salad or coleslaw, and 4 drinks - juice, coke, ginger beer or water. For dinner I decide to have 1 pizza, 1 side, and 1 drink. How many possible meals do I have to choose from?

2. How many different car number plates can be made if each is to display 3 letters followed by 3 numbers?

3. Your friend wants to perform a magic trick and asks you to draw 2 cars from a standard deck of 52. The first card you draw must be placed face down and the second placed face up on the table. How many ways are there of drawing the 2 cards?

For any counting number (n),
$n! = n(n-1)(n-2)\ldots 1$
$0! = 1$
$1! = 1$

8! This is read as 8 factorial which means $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$. We can evaluate this and see that it is equal to 40,320.

On your Calculator, the factorial symbol (!) is located at **MATH, PRB, # 4.

Examples:

1) Evaluate 4!

2) Evaluate 5

3) Evaluate 6!

4) Evaluate 7!

5) Jean is making sandwiches for a class picnic. She is using 4 different fillings with 2 different kinds of bread. How many different kinds of sandwiches can she make using one kind of filling on one kind of bread for each sandwich?

6) On a restaurant menu, there are six sandwich choices and three beverage choices. How many different lunches may a person order consisting of one sandwich and one beverage?

7) John has 6 pairs of pants and 3 shirts. How many possible outfits consisting of one shirt and one pair of pants can he select?

8) Three CD's will be selected from a collection to be played at a party. The collection has 2 hip-hop CD's, 3 alternative CD's, 1 country CD, 2 Jazz CD's, and 2 Pop CD's from which to choose. How many different combinations of CD's can be played?

9) Josh has 6 shirts and 6 pairs of pants in his closet. Two pairs of pants have a checkered pattern, three shirts have stripes, and all the other items are solid colors. If Josh will not wear stripes and checkered patterns together, how many different shirt and pants combinations can Josh wear?

Permutations:

________________ - an arrangement of objects in some specific ____________. Permutations are especially useful when the _____________ of the data is important.

Counting rule for permutations

The number of ways to *arrange in order* n distinct objects, taking them r at a time, is

$$P_{n,r} = \frac{n!}{(n-r)!}$$

where n and r are whole numbers and $n \geq r$. Another commonly used notation for permutations is nPr.

We can calculate permutations in the calculator. Press MATH, scroll right to PRB, select #2 (nPr).

Examples:

1) Evaluate ${}_7P_3$.

2) Evaluate ${}_4P_3$.

3) Evaluate ${}_9P_2$.

4) Evaluate ${}_8P_4$.

5) Evaluate ${}_{10}P_3$.

6) How many different 4-letter arrangements can be formed using the letters of the word "JUMP", if each letter is used only once?

7) How many different five-digit numbers can be formed from the digits 1, 2, 3, 4, and 5 if each digit is used only once?

8) How many different 6-letter arrangements can be formed using the letters in the word "ABSENT", if each letter is used only once?

9) All seven-digit telephone numbers in a town begin with 245. How many telephone numbers may be assigned in the town if the last four digits do not begin or end in a zero?

For words with _________________ letters:
We use a special formula:

TEACHER

CAFETERIA

Examples:

1) MISSISSIPPI

2) DELAWARE

3) MASSACHUSETTS

4) ALASKA

5) FLORIDA

Combinations:

__________________________ – an arrangement of objects in which the order ______________________________________.

In combination problems, order is _______ taken into consideration. Therefore, the difference between permutations and combinations is that in permutations we are considering groupings and in combinations we are considering only the number of ______________________________________.

Counting rule for combinations

The number of *combinations* of n objects taken r at a time is

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

where n and r are whole numbers and $n \geq r$. Other commonly used notations for combinations include nCr and $\binom{n}{r}$.

We can calculate combinations in the calculator. Press MATH, scroll right to PRB, select #3 (nCr).

Examples:

1) Evaluate ${}_7C_3$.

2) Evaluate ${}_9C_3$.

3) Evaluate ${}_{10}C_2$.

4) Evaluate ${}_8C_6$.

5) Evaluate ${}_4C_3$.

6) Find the number of combinations of 6 things taken 3 at a time.

7) How many different committees of 3 people can be chosen from a group of 9 people?

8) A coach selects players for a team. If the coach pays no attention to the positions individual play while making the first selection, how many teams can be formed if 14 candidates try out and the coach selects 5 players?

9) A DJ has 25 songs, but has time to play only 22 on the air. How many groups of 22 songs can be selected?

Summary:

How to determine the number of outcomes of an experiment

1. If the experiment consists of a series of stages with various outcomes, use the multiplication rule or a tree diagram.
2. If the outcomes consist of ordered subgroups of r items taken from a group of n items, use the permutations rule, $P_{n,r}$.

$$P_{n,r} = \frac{n!}{(n-r)!}$$

3. If the outcomes consist of non-ordered subgroups of r items taken from a group of n items, use the combinations rule, $C_{n,r}$.

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

4.3 Homework

1) Four wires (red, green, blue, and yellow) need to be attached to a circuit board. A robotic device will attach the wires. The wires can be attached in any order, and the production manager wishes to determine which order would be fastest for the robot to use. Use the multiplication rule of counting to determine the number of possible sequences of assembly that must be tested. (*Hint:* There are four choices for the first wire, three for the second, two for the third, and only one for the fourth.)

2) Barbara is a research biologist for Green Carpet Lawns. She is studying the effects of fertilizer type, temperature at time of application, and water treatment after application. She has four fertilizer types, three temperature zones, and three water treatments to test. Determine the number of different lawn plots she needs in order to test each fertilizer type, temperature range, and water treatment configuration.

3)

Compute $P_{5,2}$.

Compute $C_{5,2}$.

Compute $P_{9,9}$.

Compute $C_{8,8}$.

4) There are three nursing positions to be filled at Lilly Hospital. Position 1 is the day nursing supervisor; position 2 is the night nursing supervisor; and position 3 is the nursing coordinator position. There are 15 candidates qualified for all three of the positions. Determine the number of different ways the positions can be filled by these applicants.

5) In the Cash Now lottery game there are 10 finalists who submitted entry tickets on time. From these 10 tickets, three grand prize winners will be drawn. The first prize is one million dollars, the second prize is one hundred thousand dollars, and the third prize is ten thousand dollars. Determine the total number of different ways in which the winners can be drawn. (Assume that the tickets are not replaced after they are drawn.)

6) The University of Montana ski team has five entrants in a men’s downhill ski event. The coach would like the first, second, and third places to go to the team members. In how many ways can the five team entrants achieve first, second, and third places?